What Models Say

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An example

$$M = \Big\langle \{a, \langle a, a\rangle\}, \{\langle a, a\rangle\} \Big\rangle$$

- What does M say?
  1. There are two things, and there is a relation that holds between the first thing and itself.
  2. There are two things, and there is a property that holds of the second thing.
- The mathematical structure is ambiguous, and so it doesn't say anything.
  - Perhaps we should consider (1) and (2) to be "predicate precisifications" of M?

Using models

- Fact: Contemporary physics uses (mathematical) models to represent physical reality.
- Truism: Physicists often can explain in words how they intend for their mathematical models to represent reality.
  - When they cannot, then they are instrumentalists.
  - Does high energy physics show that discursive understanding is no longer the goal of model building?
  - Disclaimer: I'm not qualified to speak about normal practice in physics.

Using models

- From language to model: physicists use language to specify the relevant class of models and/or to identify the representational relevant parts of models.
- From model to language: physicists use models to make statements about the physical world.

Using models

- Semantic view: To accept a theory is to believe that one if its models represents the intended object or system.
- "M represents reality" is not specific enough.
    - Example: "42 represents reality."
    - Example: "$\mathbb{R}$ represents reality."

Artifact versus Content

"It's fine to construct models with artifacts. But there must always be some way of describing the phenomenon in question that (in some sense) lacks artifacts. There must be some way of saying what is really going on. For example, although we can model mass with real numbers, there must be some underlying artifact-free description, such as the $\succeq$ and C description, from which one can recover a specification of which numerical models are acceptable, and a specification of which features of the models are artifacts." (Sider 2020, p 192)

Features of the wavefunction

"On the Ghirardi-Rimini-Weber (GRW) theory (or, for that matter, on any theory of collapse), the world will consist of exactly one physical object—the universal wave function. What happens, all that happens, is that the function changes its shape in accord with the theory's dynamical laws."

Features of the wavefunction

- Which **features** of a wavefunction belong to its representational content, and which features are representational artifacts?
- Wavefunction realists should grant that some features of the wavefunction are artifacts.
  - "ψ is written in computer modern font" is an artifact.
  - "$\{\emptyset\}$ is in the domain of ψ" is an artifact.

Features of the wavefunction

- Real features of ψ
  - Expectation values
    - But: even expectation values are not invariant under all isomorphisms, e.g. the isomorphism that takes "position is represented by the operator Q" to "position is represented by the operator $Q + aI$".
- Pseudo-features of ψ
  - ψ is a subset of $\mathbb{R}^{3n} \times \mathbb{C}$, and there is a $c \in \mathbb{C}$ such that $\langle \emptyset, c \rangle \in \psi$.
  - ψ is represented by "ψ"

Features of the wavefunction

- Possibly real features of ψ
  - The value of ψ at a particular point $a \in X$
    - What about Lebesque measure zero sets?
    - What about $U(1)$ gauge freedom?
  - The value of $d\psi/dx$ at a particular point $a \in X$

Features of a group

- Real features of G
  - Cardinality
  - Abelian or non-abelian
  - Cyclic of order n
- Pseudo-features of G
  - G contains some particular element a.
  - $G = G$ and the continuum hypothesis is true.

Features of a group

- Real features of G: liberalized
  - G has n normal subgroups.
  - G has 2 irreducible representations.

Features of a spacetime

- Pseudo-features of (M, g_{ab})
  - The value of the scalar curvature at a particular point $a \in M$
- Real features of (M, g_{ab})
  - The scalar curvature has no upper bound.
- Possibly real features of (M, g_{ab})
  - Inextendible?

Note

- Do physicists sometimes take models with different features to be representationally equivalent?
- Yes, if we take "features" in a liberal sense.
  - Example: distinct groups that are isomorphic
- Is there a more refined sense of "features" such that models are representationally equivalent only if they have the same features?
  - Example: if T_1 and T_2 are Morita equivalent, then for each model M of T_1 there is a **Morita twin** model N of T_2.
  - Example: Hamiltonian and Lagrangian mechanics
  - Example: Spacetimes and Einstein algebras

Proposals

- If a property Φ of M is real, then Φ is mathematical (in some sense to be made precise).
- If a property Φ of M is real, then Φ is invariant under isomorphisms.
    - Problem: Which isomorphisms? Two mathematical objects can be isomorphic relative to one category and non-isomorphic relative to another category.
        - Example: Minkowski versus FRW spacetime.

Overview

- Even for a first-order theory T, it's not clear what counts as a genuine feature of a model M of T.
  - Signature-free accounts
    - Set-theoretic
    - Category-theoretic
  - Signature-relative accounts
    - Elementary properties
    - Second-order properties
    - ⋮
    - Väänänen's sort logic (designed to account for relational properties with other mathematical objects)

Set-theoretic properties of models

- Suppes' proposal: If Φ is a predicate in the language of ZF set theory such that $ZF \vDash \Phi(M)$, then Φ represents a property of M.
- Reply: Physicists don't care about most set-theoretic properties.
  - Physicists aren't concerned about whether M contains $\emptyset$, or $\{\emptyset\}$, or $\{\{\emptyset\}\}$, etc.
  - "$M = M$ and the continuum hypothesis is true" is an isomorphism-invariant property of M.
    - The problem is not that Φ is extrinsic to M, because physicists do care about some extrinsic properties of their models!

Category-theoretic properties of models

- Idea 1: A model's relations to other models is an important aspect of its use in physics.
  - Example: Inextendible spacetimes
    - But is it inextendible because of something about its internal structure?
- Idea 2: A model's symmetries are an important aspect of its use in physics.
  - Example: In EM, how we understand the content of a model depends on what we take to be isomorphisms between models.

Category-theoretic properties of models

- Category theory is a first-order theory with two sort symbols O and A, two function symbols $d_0, d_1 : A \to O$, a function symbol $1 : O \to A$, and a partial function symbol $\circ : A \times A \to A$.
- A model $\mathbf{C}$ of category theory consists of two sets $\mathbf{C}_0$ and $\mathbf{C}_1$.
- If we take a property of $a \in \mathbf{C}_0$ to be a sentence $\phi(x)$ such that $\mathbf{C} \models \phi(a)$, then the typical property of objects are relational features such as:
  - Being embeddable in certain other kinds of objects.
  - Having a certain number of automorphisms.

Signature-dependent accounts

- Let Σ be a signature, and let M be a Σ-structure.
- FO sentences yield properties: M has property ϕ iff $M \vDash \phi$.
  - Example: "M has two elements" is true just in case $M \vDash \exists_{=2}(x = x)$.
- Fact: These elementary properties are isomorphism invariant.

Signature-dependent accounts

- Typically there are isomorphism-invariant properties that are not expressible by first-order sentences.
- Some are expressible by second-order sentences.
  - Example: "M is uncountably infinite."
  - Example: "M is a compact topological space."
- Some are not expressible by second-order sentences.
  - Example: "M can be embedded in $\mathbb{R}^4$."

Tentative conclusions

- There are too many set-theoretic properties of models.
- There are too few category-theoretic properties of models.
- Any good account of the properties of models will make these properties signature-dependent.

Application to examples

- Relativity: Lorentzian geometry may not be first-order, but there is a sense in which it is a linguistically formulated theory.
  - There is progress to be made on isolating the relevant features of models.
- Quantum: Any two wavefunctions are related by a unitary symmetry. Thus, by standard accounts, no wavefunction can have features that another lacks.
  - What wavefunction realists need is an account of basis-relative features — i.e. features of elements of $L_2(X)$ or $\ell_2(X)$.

Conclusions

- I think that "artifact free representation" is a will o' the wisp.
  - The goal of my suggested "program" is not — like Hartry Field's — to eliminate representational artifacts from models.
  - The goal is rather to understand the subtle practice of distinguishing relevant (intended) from irrelevant (unintended) features of models.
- The dilemma for language-free accounts of the features of models:
  - If models are sets, then they have more properties than physics cares about.
  - If models are objects in a category, then they have fewer properties than physics cares about.

Conclusions

- Believing that M has artifacts is tantamount to rejecting the unqualified statement that M represents the world.
- We use M to generate sentences that describe the world.
- Perhaps mathematics first, but not only mathematics.