

What is a deterministic theory?

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Who cares about determinism?

- Determinism as the enemy of human freedom and responsibility
- Determinism as the friend of scientific understanding, explanation, intelligibility, and control

Determinism and natural philosophy

“Determinism wins our unceasing admiration in forcing to the surface many of the more important and intriguing issues in the length and breadth of the philosophy of science.” (Earman, *Primer of Determinism*, p 21)

- Hole argument
- Determinism
- Syntactic versus semantic view of theories
- Formal definitions (e.g. of equivalence)

Negative perspectives

- Unclear: Determinism is an ontological thesis (Earman 1986)
- Deleterious: Determinism is not a formal property of theories, but is a property of interpreted theories (Belot 1995)
- False dilemma: Determinism is a matter of semantics rather than syntax

Positive perspectives

- Arrows between models are an essential part of the content of a theory
 - Sometimes clarification calls for formal specification
- Let empirical science (physics) bend and stretch our metaphysical frameworks
- Syntax as an aid in the quest for clarity

Definitions of determinism

- Syntactic
 - Smart
 - Nagel
- Semantic
 - Montague
 - Lewis
 - Earman
 - Butterfield
 - Dm1
 - Dm2
 - Landsman

What's wrong with syntactic analyses?

1. Montague: For each time t , there is a distinct sentence ϕ_t that is true at t . But then there are uncountably many sentences, which is not permitted in “standard” languages.
2. Earman: If we have to use an uncountably infinite signature, then the completeness theorem fails.

Earman against syntactic analyses

“Many philosophical discussions of determinism are couched in terms of theories, construed as linguistic entities. But since determinism is a doctrine about the nature of the world, no problem is avoided by this linguistic detour.” (PD, p 20)

Belot against formal analyses

“In the philosophical literature, there are two common criteria for a physical theory to be deterministic. The older one is due to the logical empiricists, and is a purely formal criterion. The newer one can be found in the work of John Earman and David Lewis and depends on the intended interpretation of the theory. In this paper I argue that the former must be rejected, and something like the latter adopted.” (Belot 1995, p 85)

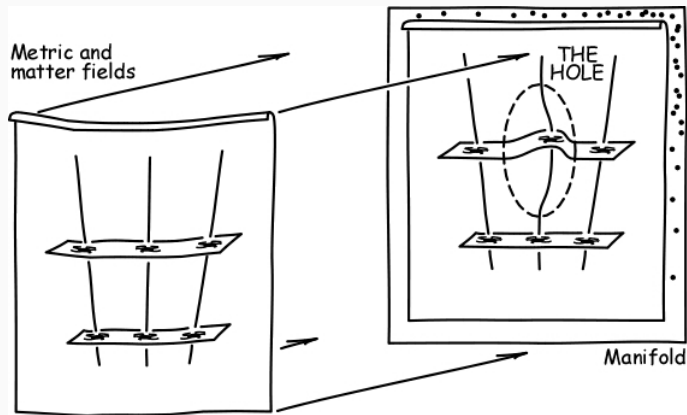
Belot against formal analyses

“I argue that you cannot decide whether or not a theory is deterministic without knowing something about the interpretation of the theory. Thus determinism cannot be a formal property of theories. On the other hand, providing an interpretation for a theory naturally involves dealing with possibilities.” (Belot 1995, p 85)

A world $W \in \mathcal{W}$ is *Laplacian deterministic* just in case for any $W' \in \mathcal{W}$, if W and W' agree at any time, then they agree for all times.

“It might be charged that the possible worlds analysis is a fraud: it is no more than a transcription of James’ poetic vision into terms which are devoid of James’ eloquence but which display not much compensating gain in clarity and precision. I couldn’t agree more! But I also think that without prejudging detailed substantive issues in physics we cannot do much better for a direct ontological formulation of what is, after all, an ontological doctrine.”
(Earman, PD, p 14)

The hole argument



The most boring theory ever

- Predicates and relations: None
- Axiom: $\exists!x(x = x)$

T is indeterministic (if the possible worlds definition is read literally)

- Dm1** For any models M and N , and for any diffeomorphism $\varphi : M \rightarrow N$, if φ restricts to an isometry between initial segments of M and N , then $\varphi : M \rightarrow N$ is an isometry.
- Dm2** For any models M and N , if there is an isometry α between initial segments of M and N , then there is an isometry $\beta : M \rightarrow N$.

State of play

- Belot (1995) and Melia (1999) give examples that are Butterfield-Lewis deterministic, but are intuitively indeterministic.
 - Suggestion that Lewis-Butterfield determinism is a cheap, second-rate kind of determinism
 - But the super-strong definitions have it that most physical theories are indeterministic, and especially GTR.
- The myth persists that there is a “metaphysically deeper” form of determinism.
- This myth has given rise to a false dilemma between “formal” and “metaphysical” responses to the hole argument.

“Some, such as Earman and Norton ([1987]) and Belot ([1995]), believe that such differences between possibilities *should* be counted as relevant to determinism; others, such as Lewis ([1983]), Butterfield ([1989]) and Brighouse ([1994], [1997]), believe that such differences should be ignored.” (Melia 1999, p 640)

Belot and Melia's examples

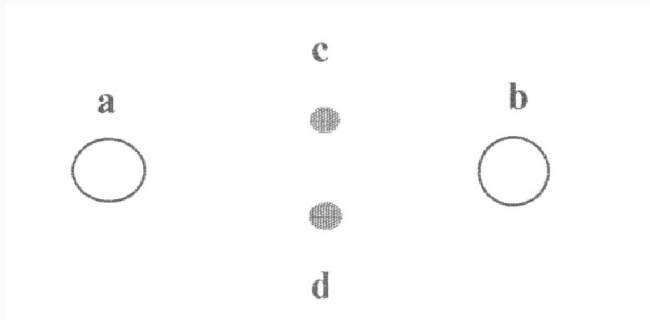
- Buckling column
- Bald philosophers

“This example gives us a recipe for generating distinct possibilities whose futures differ only over which objects play which roles.”

At time t , there are several objects which share not only their intrinsic qualitative properties, but their relational ones as well.

At some time t^* later than t , one of these duplicates changes one of its intrinsic or extrinsic properties.

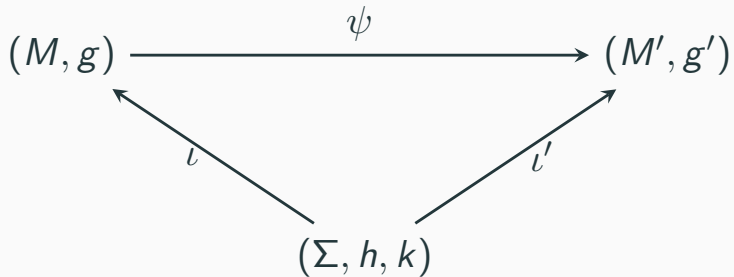
Melia's examples



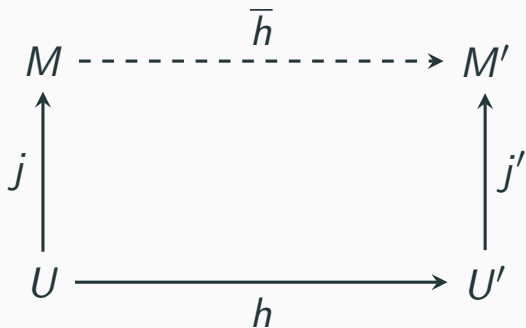
Manchak-Halvorson: Property R

$$\begin{array}{ccc} M & \begin{array}{c} \xrightarrow{f} \\ \xrightarrow{g} \end{array} & N \\ \uparrow j & & \uparrow j' \\ M|_U & \xrightarrow{f|_U = g|_U} & N|_U \end{array}$$

Landsman determinism



Butterfield-Landsman determinism



From semantic to syntactic analysis

- Claim: Let M, M' be models of T , and let $h : M \rightarrow M'$ be a bijection that preserves (i) linking functions, and (ii) restricted vocabulary. Then h preserves extensions of full vocabulary.
- Beth's Theorem: Full vocabulary is definable in terms of restricted vocabulary and linking functions.
- All facts are deducible from facts about an initial segment.

How determinism can fail

1. An isomorphism of initial data does not extend to an isomorphism of models.
 - Models have more structure than the initial data.
 - Spontaneous symmetry breaking.
2. An isomorphism of initial data has more than one distinct extension an isomorphism of models.

MiniMach spacetime

Assumptions:

- Two times, two identical spatial points at each time
 - No relations between points at different times
-

T is categorical (one model up to isomorphism)

Initial conditions have a unique extension to a spacetime model.

But an isomorphism of initial conditions does not uniquely determine an isomorphism of models.

Therefore, T is indeterministic.

Bald philosophers

Assumptions:

- There are seven identical, bald philosophers.
- At a subsequent time, one of these philosophers grows a hair.

Let h be a permutation of the initial data that exchanges the philosopher who will grow a hair with another philosopher.

There is no isomorphism \bar{h} that extends h to the entire model.

Therefore, T is indeterministic.

White and black spheres

Assumptions:

- There are two identical white spheres and two identical black spheres
- Each white sphere pairs up with a black sphere.
- Hence there is a bijection f from white spheres to black spheres.

Let h be the automorphism of the domain that fixes the white spheres, but flips the black spheres.

There is no isomorphism \bar{h} that extends h to the entire model.

Therefore, T is indeterministic.

- We appear to have a good definition. It explains the difference between GTR (deterministic) and the examples by Belot and Melia (indeterministic).
- We don't need haecceitism to judge that Belot and Melia's examples are indeterministic.
- Theories with one model (up to isomorphism) might be indeterministic.

First Order Lessons

- Theories with unique final conditions (up to isomorphism) might be indeterministic.

“Let us say that *D-haecceitism* is the view that a theory may be indeterministic, even if all the different possible futures open to any world which makes the theory true are qualitatively identical.”

(Melia 1999, p 640)

- Determinism isn't a matter of identity between worlds or world segments.
- A theory is deterministic if change of representation of initial segments determines a change of representation of entire histories.