Logic pset 15

Resources: HLW Ch 8 "Models" and Lecture 20

1. For each of the following sequents, provide a counterexample to show that it is invalid.

   (a) $\forall x Fx \rightarrow \forall x Gx \vdash \forall x(Fx \rightarrow Gx)$

   (b) $\forall x(Fx \rightarrow Gx) \vdash \exists x(Fx \wedge Gx)$

   (c) $\vdash \forall x Fx \vee \forall x \neg Fx$

   (d) $\exists x(Fx \rightarrow P) \vdash \exists x Fx \rightarrow P$

2. For each of the following sentences, provide one interpretation in which it is true and one interpretation in which it is false. An interpretation may be presented by giving a set M and a subset R^M of $M \times M$, or it may be presented as an arrow diagram.

   (a) $\forall x \forall y \exists z(Rxz \wedge Ryz)$

   (b) $\forall x(\exists y Ryx \rightarrow \forall z Rzx)$