

Translations between Translations

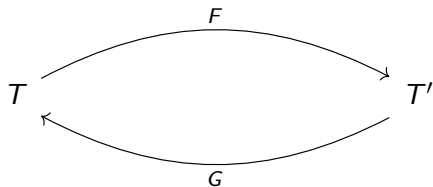
Hans Halvorson

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In Celebration of István Németi's 80th Birthday

When philosophers decided that rigorous methods weren't worth the effort, István Németi kept the tradition alive

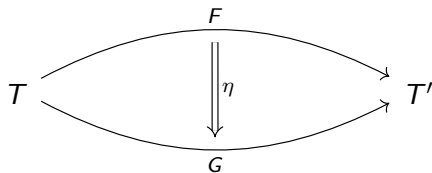
Standard definition of equivalence for theories



$$T \vdash \phi \leftrightarrow GF(\phi)$$

$$T' \vdash \psi \leftrightarrow FG(\psi)$$

Arrows between translations



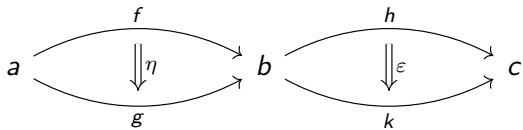
$$T' \vdash F(\phi) \rightarrow G(\phi)$$

$$\eta : 1_T \Rightarrow GF$$

$$\varepsilon : FG \Rightarrow 1_{T'}$$

Propositional theories form a “poset enriched category”

2-categories



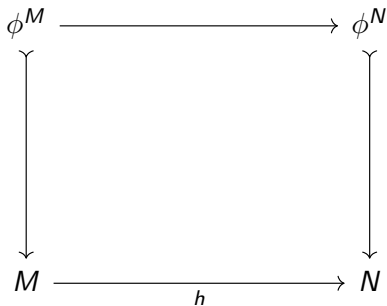
Example: 2-category of categories

0-cells categories

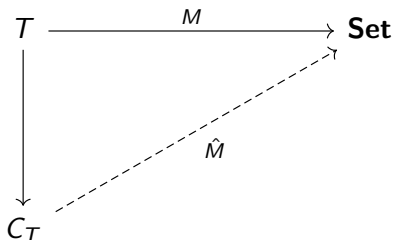
1-cells functors

2-cells natural transformations

Arrows between models: elementary embedding and symmetry



Models as functors



Elementary embeddings (in particular, model automorphisms) are 2-cells

Philosophical questions

- ▶ What is a good notion of translation between theories?
 - ▶ When are two translations “the same”?
 - ▶ What is the meaning of a morphism between translations?
- ▶ What is a good notion of **equivalence** between theories?
- ▶ What is a good notion of **reduction** of one theory to another?

2-categorical philosophy

- ▶ Abstract objects have no principle of individuation
- ▶ Question is not “are x and y equivalent?” but “in which ways is x shown to be equivalent to y ?”
 - ▶ This applies to theories in particular
 - ▶ This applies when y is replaced with x
- ▶ Questions of the form “how many ϕ are there?” are ill defined
 - ▶ Two objects with one isomorphism between them is a different situation than two objects with two isomorphisms between them

∞ -categorical philosophy?

If 0-cells cannot be individuated, then why think that 1-cells can be individuated?

If 1-cells cannot be individuated, then why think that 2-cells can be individuated?

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